⑥

4(b)

$$\mathcal{L}^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\} = \mathcal{L}^{-1}\left\{\frac{s-5}{(s+2)^2+16}\right\} \qquad 1$$

$$= \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+16}\right\} + \mathcal{L}^{-1}\left\{\frac{-7}{(s+2)^2+16}\right\} \qquad 2$$

$$= e^{-2t}\cos(4t) - \frac{7}{4}e^{-2t}\sin(4t) \equiv f(t) \qquad 3$$

4(c) $\quad \cancel{6}$

$$\therefore \mathcal{L}^{-1}\left\{\frac{e^{2s}(s-5)}{s^2+4s+20}\right\} = f(t+2)\,\mathcal{U}(t+2) \qquad 2$$

$$= e^{-2t-4}\,\mathcal{U}(t+2)\left\{\cos(4t+8) - \frac{7}{4}\sin(4t+8)\right\} \qquad 2$$

$\cancel{4}$

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5. $\quad y'' + 4y' + 4y = 12t+6 \qquad y(0) = -2 \quad y'(0) = 4$

$$\mathcal{L}(y'') + 4\mathcal{L}(y') + 4\mathcal{L}(y) = \mathcal{L}(12t) + \mathcal{L}(6) \qquad 1$$

$$\Rightarrow s^2Y - \cancel{sy(0)} - y'(0) + 4\{sY - y(0)\} + 4Y = \frac{12}{s^2} + \frac{6}{s} \qquad 3$$

$$\Rightarrow s^2Y + 2s - 4 + 4sY + 8 + 4Y = \frac{12}{s^2} + \frac{6}{s} = \frac{6(s+2)}{s^2} \qquad 2$$

$$\Rightarrow Y[s^2+4s+4] + 2s + 4 = \frac{6(s+2)}{s^2} \qquad 1$$

$$\text{i.e. } Y(s+2)^2 = \frac{6(s+2)}{s^2} - 2(s+2) \qquad 1$$

$\Rightarrow$