7(b) $9\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$. Let $u = X(x)T(t)$. Sub into

d.e. $\Rightarrow \quad \frac{X''}{X} = \frac{T'}{9T} = -\alpha^2$ 1

$\Rightarrow X'' + \alpha^2 X = 0 \quad , \quad T' + 9\alpha^2 T = 0$ 1

Soln $X = A\cos(\alpha x) + B\sin(\alpha x)$; $T = Ce^{-9\alpha^2 t}$ 2

$u(0,t) = X(0)T(t) = 0 \Rightarrow X(0) = 0 = A$ 1

$u(2\pi,t) = X(2\pi)T(t) = 0 \Rightarrow X(2\pi) = B\sin(2\pi\alpha) = 0$ 1

$\Rightarrow 2\pi\alpha = n\pi$ i.e. $\boxed{\alpha = \frac{n}{2}}$, $n = 1, 2, 3, \ldots$ 2

($\alpha = 0$ will not contribute)

$\therefore \quad u = B\sin\left(\frac{nx}{2}\right) Ce^{-9n^2 t/4}$

Go to infinite sum & Fourier series

$$u = \sum_1^\infty b_n \sin\left(\frac{nx}{2}\right) e^{-9n^2 t/4}$$ 1

Apply I.C.

$$u(x,0) = \sum_1^\infty b_n \sin\left(\frac{nx}{2}\right) = f(x)$$ 1

This is the Fourier sine series $\Rightarrow$ b_n is given in (a) i.e. $b_n = \frac{8}{\pi n^2}(-1)^m$ if $n = 2m+1$ (n odd)

$= 0$ if n is even 2