4. $x^2y'' - 2xy' + 2y = x^3 \ln x$

Homog. eqn: $x^2y'' - 2xy' + 2y = 0 \longrightarrow m(m-1) - 2m + 2 = 0$

$\Rightarrow m^2 - 3m + 2 = (m-1)(m-2) = 0 \Rightarrow m = 1, 2.$ 2

$\therefore y_c = c_1 x + c_2 x^2$; $f(x) = x \ln x$. 2

Using variation-of-parameters $y_p = v_1 x + v_2 x^2$. 1

$$\left.\begin{array}{l} v_1' x + v_2' x^2 = 0 \\ v_1' + 2v_2' x = x \ln x \end{array}\right\} \Rightarrow \left.\begin{array}{l} v_1' = -x v_2' \\ -x v_2' + 2v_2' x = x \ln x \end{array}\right\} \begin{array}{l} v_1' = -x v_2' \\ v_2' = \ln x \end{array}$$ 4

$\therefore v_2 = x \ln x - x \Rightarrow v_1' = -x \ln x \Rightarrow v_1 = \frac{-1}{2} x^2 \ln x + \frac{x^2}{4}$ 4

Thus

$$y = c_1 x + c_2 x^2 + \left\{ -\frac{x^2 \ln x}{2} + \frac{x^2}{4} \right\} x + \left\{ x \ln x - x \right\} x^2$$ 1

$$= c_1 x + c_2 x^2 + \frac{1}{2} x^3 \ln x - \frac{3}{4} x^3$$

$$\boxed{y = c_1 x + c_2 x^2 + \frac{x^3}{2}\left[\ln x - 3/2\right]}$$ 1

14

is the gen. soln.

Alternatively (using the Wronskian) $y_1 = x$, $y_2 = x^2$.

$$W = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 = x^2$$