8(a)

$$z = \left(\frac{1+i}{1-i}\right)^{i/2} = \left[\frac{(1+i)^2}{2}\right]^{i/2}$$ 1

$$\left(\frac{1+i}{2}\right)^2 = \frac{1}{2}(1-1+2i) = i$$ 1

$$\therefore z = [i]^{i/2} = e^{\ln(z)}$$ 1

$$\ln(z) = \frac{i}{2}\ln(i) = \frac{i\cdot i}{2}\left(\frac{\pi}{2} + 2n\pi\right) = -\left(\frac{\pi}{4} + n\pi\right)$$ 2

Thus $z = e^{-(\pi/4 + n\pi)} = e^{-\pi(1/4+n)}$, n integer 2

8(b) $\cosh z = 5i$ 7

$\Rightarrow \cosh x\cos y + i\sinh x\sin y = 5i$ 1

$\Rightarrow \cosh x\cos y = 0 \ldots(1)$, $\sinh x\sin y = 5 \ldots(2)$ 1

(1) $\Rightarrow \cos y = 0 \Rightarrow \boxed{y = \frac{(2n+1)\pi}{2}}$ (since $\cosh x \neq 0$) 2

sub this result into (2) $\Rightarrow \sinh x \sin\left[\frac{(2n+1)\pi}{2}\right] = 5$ 1

$\Rightarrow \sinh x\,(-1)^n = 5 \Rightarrow x = \sinh^{-1}[(-1)^n 5]$ 2

Soln $z = \sinh^{-1}[(-1)^n 5] + \frac{i\pi}{2}(2n+1)$. 1

(15) 8