

Instructions. This paper contains 9 questions with total marks 163. The mark allocated to each question is indicated. Full credit may be obtained by achieving 135 marks.

1. Solve the following initial -value problems.

(a). $x(x-1)y' + 2xy = 1, \quad y(2) = 2.$

(b). $2(y+xy)y' + x + xy = 0, \quad y(0) = 1. \quad [8 + 8=16 \text{ Marks}]$

2. A tank contains 100 litres of water in which 20 Kg of salt is dissolved. At $t = 0$ a salt solution containing 2 Kg of salt per litre flows into the tank at the rate of 4 litres per minute. The liquid in the tank is kept homogeneous by constant stirring. Also, at $t = 0$, liquid begins to flow from the tank at the rate of 2 litres per minute. The tank holds 150 litres.

- (a). Determine the amount of salt in the tank at any time t before overflow.
(b). How much salt is in the tank at the end of 5 minutes?
(c). What is the concentration of salt in the tank at the instant the tank overflows?

[15+2+3=20 Marks]

3. Solve the following initial-value problem.

$$y'' - 4y' + 4y = 4x + 5 \cos(x); \quad y(0) = 1, y'(0) = -1$$

[18 Marks]

4. (a). Determine $\mathcal{L}\{e^{-4t} \cos(2t) + \int_0^t \tau \sin(\tau) d\tau\}$, where \mathcal{L} represents the Laplace transform operator.

(b). Determine $\mathcal{L}^{-1}\{(s^2 - 6s - 9)/[(s^2 + 9)(s - 3)]\}$

(c). Evaluate $\mathcal{L}\{u(t-1) * [te^{-2t}]\}$, where $u(t)$ is the unit step function, using the convolution theorem.

[7+7+5=19 Marks]

5. Using the Laplace transform, solve the initial-value problem

$$y'' + y' - 6y = 2e^{3x}, \quad y(0) = 1, \quad y'(0) = 4.$$

[15 Marks]

6. Find the first 6 non-zero terms of the power series solution about $x = 0$, of the following initial-value problem.

$$y'' + xy' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 4$$

[14 Marks]