8(b) $u = XT \Rightarrow 4X''T = XT'' \Rightarrow \dfrac{X''}{X} = \dfrac{T''}{4T} = -\lambda^2$

$\Rightarrow X'' + \lambda^2 X = 0 , \quad T'' + 4\lambda^2 T = 0$, 2

*Now for $\lambda \neq 0$

$\Rightarrow X = c_1 \sin(\lambda x) + c_2 \cos(\lambda x)$; $T = c_3 \sin(2\lambda t) + c_4 \cos(2\lambda t)$ 2

$u(0,t) = 0 \Rightarrow X(0) = 0 \Rightarrow c_2 = 0$ 1

$u(2\pi,t) = 0 \Rightarrow c_1 \sin(2\pi\lambda) = 0 \Rightarrow \sin(2\pi\lambda) = 0 , \; c_1 \neq 0$ 2

$\Rightarrow 2\pi\lambda = n\pi \Rightarrow \boxed{\lambda = \dfrac{n}{2}} , \; n = 0, 1, 2, \ldots$ 2

$$\frac{\partial u}{\partial t} = c_1 \sin(\lambda x)\left[2\lambda c_3 \cos(2\lambda t) - 2\lambda c_4 \sin(2\lambda t)\right]$$

$\Rightarrow \dfrac{\partial u}{\partial t}(x,0) = 0 \Rightarrow 2\lambda c_3 = 0 \Rightarrow \boxed{c_3 = 0}$ 2

(* $\lambda = 0$ leads to $u \equiv 0$)

$u = c_1 \sin(\lambda x) c_4 \cos(2\lambda t) \equiv b \sin(\lambda x) \cos(2\lambda t)$ 1

Infinite sum:

$$u = \sum_{1}^{\infty} b_n \sin\left(\frac{nx}{2}\right) \cos(nt)$$ 1

$$u(x,0) = \sum_{1}^{\infty} b_n \sin\left(\frac{nx}{2}\right) = f(x)$$ 1

Hence b_n are the Fourier sine coefficients 2

as computed in 8(a).

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