8(a)
$$f(x) = \begin{cases} x/\pi, & 0 < x \le \pi \\ 2 - \dfrac{x}{\pi}, & \pi < x < 2\pi \end{cases}$$

$$a_n = \frac{2}{2\pi}\int_0^{2\pi} f(x)\sin\left(\frac{n\pi x}{2\pi}\right)dx \qquad (n = 1, 2, 3, \ldots)$$ 1

$$= \frac{1}{\pi^2}\int_0^{\pi} x\sin\left(\frac{nx}{2}\right)dx + \frac{1}{\pi}\int_{\pi}^{2\pi} 2\sin\left(\frac{nx}{2}\right)dx$$

$$- \frac{1}{\pi^2}\int_{\pi}^{2\pi} x\sin\left(\frac{nx}{2}\right)dx$$ 2

$$= \frac{1}{\pi^2}\left\{\frac{2^2}{n^2}\sin\left(\frac{nx}{2}\right) - x\frac{2}{n}\cos\left(\frac{nx}{2}\right)\right\}_0^{\pi}$$

$$+ \frac{2}{\pi}\left\{-\frac{2}{n}\cos\left(\frac{nx}{2}\right)\right\}_{\pi}^{2\pi} - \frac{1}{\pi^2}\left\{\frac{4}{n^2}\sin\left(\frac{nx}{2}\right) - \frac{2x}{n}\cos\left(\frac{nx}{2}\right)\right\}_{\pi}^{2\pi}$$ 5

$$= \frac{1}{\pi^2}\left\{\frac{4}{n^2}\sin\left(\frac{n\pi}{2}\right) - \frac{2\pi}{n}\cos\left(\frac{n\pi}{2}\right)\right\}$$

$$+ \frac{2}{\pi}\left\{-\frac{2}{n}\cos(n\pi) + \frac{2}{n}\cos\left(\frac{n\pi}{2}\right)\right\} - \frac{1}{\pi^2}\left\{-\frac{4\pi}{n}\cos(n\pi)\right.$$

$$\left. - \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) + \frac{2}{n}\cos\left(\frac{n\pi}{2}\right)\right\}$$ 3

$$= \cos\left(\frac{n\pi}{2}\right)\left\{-\frac{2}{\pi n} + \frac{4}{n\pi} - \frac{2}{n\pi}\right\}$$ (0)

$$+ \sin\left(\frac{n\pi}{2}\right)\left\{\frac{4}{\pi^2 n^2} + \frac{4}{n^2\pi^2}\right\} + \cos(n\pi)\left\{\frac{-4}{n\pi^2} + \frac{4}{n\pi}\right\}$$ (0) 2