

SIT 294 Engineering Maths

Semester 1, 2006

Exam solutions

$$1(a) \quad x(x-1)y' + 2xy = 1 \Rightarrow y' + \frac{2}{(x-1)}y = \frac{1}{x(x-1)} \quad (\text{S.F.}) \quad x \neq 1$$

$$v(x) = \frac{2}{x-1} \Rightarrow \mu = e^{\int \frac{2dx}{x-1}} = e^{2\ln(x-1)} = (x-1)^2 \quad 3$$

$$\frac{d}{dx} \left\{ (x-1)^2 y \right\} = \frac{(x-1)^2}{x(x-1)} = \frac{x-1}{x} \quad 2$$

$$\therefore (x-1)^2 y = x - \ln|x| + C \Rightarrow \boxed{y = \frac{x - \ln x + C}{(x-1)^2}} \quad 2$$

$$y(2) = 2 \Rightarrow 2 = 2 - \ln 2 + C \Rightarrow C = \ln 2 \quad 1$$

$$\therefore y = \frac{x - \ln x + \ln 2}{(x-1)^2} \quad \underline{8}$$

$$1(b) \quad 2(y+xy)y' + x + xy = 0 \Rightarrow 2y(1+x)y' + x(1+y) = 0$$

$$\Rightarrow 2y(1+x)dy = -x(1+y)dx \quad (\text{separate variables})$$

$$\Rightarrow \frac{2y}{1+y} dy = \frac{-x dx}{1+x} \quad 2$$