7. $y = \sum_0^\infty a_n x^n$, $y' = \sum_0^\infty n a_n x^{n-1}$, $y'' = \sum_0^\infty n(n-1) a_n x^{n-2}$

sub. into d.e. yields

$$2\sum_0^\infty n(n-1)a_n x^{n-2} + \sum_0^\infty n a_n x^n + 2\sum_0^\infty a_n x^n = 0$$ 1

Now shift index on 1st term

$$\sum_0^\infty n(n-1)a_n x^{n-2} = \sum_0^\infty (n+2)(n+1)a_{n+2} x^n$$ 1

d.e. becomes

$$\sum_0^\infty \{(n+2)(n+1)a_{n+2} + n a_n + 2a_n\} x^n = 0$$ 1

$$\Rightarrow (n+2)(n+1)a_{n+2} + (n+2)a_n = 0$$ 1

i.e. $$a_{n+2} = \frac{-(n+2)a_n}{(n+2)(n+1)} = \frac{-a_n}{n+1}, \quad n = 0, 1, 2, \ldots$$ 2

Now $$a_2 = \frac{-a_0}{1} = -a_0,$$ 1

$$a_3 = \frac{-a_1}{2}, \quad a_4 = \frac{-a_2}{3} = \frac{a_0}{3}$$ 2

$$a_5 = \frac{-a_3}{4} = \frac{a_1}{8}, \quad a_6 = \frac{-a_4}{5} = \frac{-a_0}{15}$$ 2

$\therefore$ Gen soln is $y = a_0 + a_1 x + (-a_0)x^2 - \frac{a_1}{2}x^3 + \frac{a_0}{3}x^4 + \frac{a_1}{8}x^5 \ldots = a_0(1 - x^2 + x^4/3 \ldots) + a_1(x - x^3/2 + x^5/8 \ldots)$

1