3(b) $x^2y'' + 7xy' + 5y = \frac{16}{x}$ ; $y(1) = 2,\ y'(1) = -2$

Hom. d.e. $x^2y'' + 7xy' + 5y = 0 \rightarrow m(m-1) + 7m + 5 = m^2 + 6m + 5 = 0$

$\Rightarrow (m+5)(m+1) = 0 \Rightarrow m = -5, -1. \therefore y_c = c_1 x^{-5} + c_2 x^{-1}$ $(x \neq 0)$ 3

Now $y_1 = x^{-5}$, $y_2 = x^{-1}$ and $f(x) = \frac{16}{x^3}$ 1

$$W = \begin{vmatrix} x^{-5} & x^{-1} \\ -5x^{-6} & -x^{-2} \end{vmatrix} = -x^{-7} + 5x^{-7} = 4x^{-7}$$ 2

$$\therefore u_1 = -\int \frac{x^{-1}}{4x^{-7}} \cdot \frac{16}{x^3}\,dx = -\int 4x^3\,dx = -x^4$$ 2

$$u_2 = \int \frac{x^{-5}}{4x^{-7}} \cdot \frac{16}{x^3}\,dx = \int \frac{4}{x}\,dx = 4\ln x$$ 2

$\therefore$ Gen. soln.

$$y = c_1 x^{-5} + c_2 x^{-1} - \frac{1}{x} + \frac{4\ln x}{x}$$ 1

$$\Rightarrow y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{x^2} + \frac{4}{x^2} - \frac{4\ln x}{x^2}$$ 1

$\therefore y(1) = 2 \Rightarrow c_1 + c_2 - 1 = 2 \Rightarrow c_1 + c_2 = 3$

$y'(1) = -2 \Rightarrow -5c_1 - c_2 + 1 + 4 = -2 \Rightarrow -5c_1 - c_2 = -7$

$\Big\}$ $c_1 = 1$, $c_2 = 2$ 4