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$$6. \quad \mathcal{L}\{y'' + y' - 6y\} = 2\mathcal{L}\{e^{3x}\}, \quad y(0)=1, \quad y'(0)=4.$$

$$\Rightarrow s^2 \check{Y}(s) - s - 4 + s \check{Y}(s) - 1 - 6 \check{Y}(s) = \frac{2}{s-3} \quad 2$$

$$\Rightarrow Y(s^2 + s - 6) - s - 5 = \frac{2}{s-3} \quad 1$$

$$\Rightarrow Y(s+3)(s-2) = \frac{2}{s-3} + s+5 = \frac{2 + (s+5)(s-3)}{(s-3)} \quad 2$$

$$= \frac{2 + s^2 + 2s - 15}{s-3} = \frac{s^2 + 2s - 13}{s-3} \quad 1$$

$$\therefore Y = \frac{s^2 + 2s - 13}{(s+3)(s-2)(s-3)} = \frac{A}{s+3} + \frac{B}{s-2} + \frac{C}{s-3} \quad 1$$

$$\Rightarrow s^2 + 2s - 13 = A(s-2)(s-3) + B(s+3)(s-3) + C(s+3)(s-2) \quad 1$$

$$s=2: -5 = B(5)(-1) \Rightarrow B=1 \quad 1$$

$$s=3: 2 = 6C \Rightarrow C = 1/3 \quad 1$$

$$s=-3: -10 = +30A \Rightarrow A = -1/3 \quad 1$$

$$\therefore Y = -\frac{1}{3(s+3)} + \frac{1}{s-2} + \frac{1}{3(s-3)} \quad 1$$

$$\text{Hence } y = \mathcal{L}^{-1}(Y) = -\frac{1}{3}e^{-3t} + e^{2t} + \frac{1}{3}e^{3t} \quad 3$$

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