SCM 228 TUTORIAL 8

FOURIER SERIES of f on $(-p,p)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{p} + b_n \sin\frac{n\pi x}{p}\right]$$

where $a_0 = \frac{1}{p}\int_{-p}^{p} f(x)\,dx$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi x}{p}\,dx$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi x}{p}\,dx$$

FOURIER COSINE SERIES

of f on $(-p,p)$.

f is EVEN $\therefore b_n = 0$

$$a_0 = \frac{2}{p}\int_0^p f(x)\,dx$$

$$a_n = \frac{2}{p}\int_0^p f(x)\cos\frac{n\pi x}{p}\,dx$$

FOURIER SINE SERIES

of f on $(-p,p)$.

f is ODD $\therefore a_0 = a_n = 0$

$$b_n = \frac{2}{p}\int_0^p f(x)\sin\frac{n\pi x}{p}\,dx$$

EXAMPLES

1. Z and C 12.2 № 3 p.661.

$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ x, & 0 \le x < 1 \end{cases}$$

$$a_0 = \int_{-1}^{0} 1\,dx + \int_0^1 x\,dx$$

$$= 1 + \frac{1}{2} \qquad \therefore a_0 = \frac{3}{2}.$$

$$a_n = \int_{-1}^{0}\cos n\pi x\,dx + \int_0^1 x\cos n\pi x\,dx$$

$$\therefore a_n = \left[\frac{1}{n\pi}\sin n\pi x\right]_{-1}^{0} + \left[\frac{1}{n\pi}x\sin n\pi x + \frac{1}{n^2\pi^2}\cos n\pi x\right]_0^1$$

$$\therefore a_n = \frac{1}{n^2\pi^2}\left(\cos n\pi - 1\right)$$

$$\therefore a_n = \frac{1}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$b_n = \int_{-1}^{0}\sin n\pi x\,dx + \int_0^1 x\sin n\pi x\,dx$$

$$= \left[-\frac{1}{n\pi}\cos n\pi x\right]_{-1}^{0} + \left[\frac{1}{n^2\pi^2}\sin n\pi x - \frac{1}{n\pi}x\cos n\pi x\right]_0^1$$

$$\therefore b_n = -\frac{1}{n\pi}\left[1 - \cos(-n\pi)\right] - \frac{1}{n\pi}\cos n\pi$$

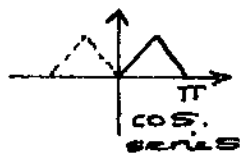
$$\therefore b_n = -\frac{1}{n\pi}.$$

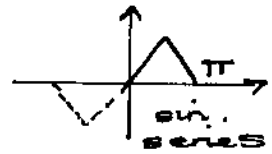
So, $f(x) = \frac{3}{4} +$

$$\sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2\pi^2}\cos n\pi x - \frac{1}{n\pi}\sin n\pi x\right]$$

2. Z and C 12.3 № 29, p.668.

$$f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 \le x < \pi. \end{cases}$$





(i) COSINE series

$$a_0 = \frac{2}{\pi}\left\{\int_0^{\pi/2} x\,dx + \int_{\pi/2}^{\pi}(\pi - x)\,dx\right\}$$

$$= \frac{2}{\pi}\left\{\left[\frac{x^2}{2}\right]_0^{\pi/2} + \left[\pi x - \frac{x^2}{2}\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{\pi^2}{8} + \pi^2 - \frac{\pi^2}{2} - \frac{\pi^2}{2} + \frac{\pi^2}{8}\right\}$$

$$= \frac{2}{\pi}\times\frac{\pi^2}{4} \qquad \therefore a_0 = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi}\left\{\int_0^{\pi/2} x\cos nx\,dx + \int_{\pi/2}^{\pi}(\pi - x)\cos nx\,dx\right\}$$

$$= \frac{2}{\pi}\left\{\left[\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi/2} + \left[\frac{\pi - x}{n}\sin nx - \frac{\cos nx}{n^2}\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{\pi}{2n}\sin\frac{n\pi}{2} + \frac{\cos n\pi/2}{n^2} - \frac{1}{n^2} - \frac{\cos n\pi}{n^2} - \frac{\pi}{2n}\sin\frac{n\pi}{2} + \frac{\cos n\pi/2}{n^2}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{2\cos n\pi/2}{n^2} - \frac{1}{n^2} - \frac{(-1)^n}{n^2}\right\}$$

$$= \frac{2}{n^2\pi}\left\{2\cos\frac{n\pi}{2} - 1 - (-1)^n\right\}.$$

EXERCISE (ii) SINE series

$$b_n = \frac{2}{\pi}\left\{\int_0^{\pi/2} x\sin nx\,dx + \int_{\pi/2}^{\pi}(\pi - x)\sin nx\,dx\right\}$$

$$= \frac{2}{\pi}\left\{\left[\frac{1}{n^2}\sin nx - \frac{x\cos nx}{n}\right]_0^{\pi/2} + \left[\frac{-(\pi - x)}{n}\cos nx - \frac{\sin nx}{n^2}\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{\sin n\pi/2}{n^2} - \frac{\pi\cos n\pi/2}{2n} + \frac{\pi\cos n\pi/2}{2n}\right\}$$

$\therefore b_n = \frac{2\sin n\pi/2}{\ldots}$ [illegible]