5(a) Consider

$$\mathcal{L}\left\{e^{-4t}\cos(2t) + \int_0^t t\sin t\, dt\right\} \equiv I$$

Now $\mathcal{L}\{\cos(2t)\} = \dfrac{s}{s^2+4} \Rightarrow \mathcal{L}\{e^{-4t}\cos(2t)\} = \dfrac{s+4}{(s+4)^2+4}$ 2

and $\mathcal{L}\left\{\int_0^t t\sin t\, dt\right\} = \dfrac{F(s)}{s}$ where $F(s) = \mathcal{L}\{t\sin t\}$ 1

$$= -\frac{d}{ds}\left\{\frac{1}{s^2+1}\right\} = \frac{2s}{(s^2+1)^2}$$ 2

$$\therefore \quad \mathcal{L}\left\{\int_0^t t\sin t\, dt\right\} = \frac{2}{(s^2+1)^2}$$ 1

Thus $I = \dfrac{s+4}{(s+4)^2+4} + \dfrac{2}{(s^2+1)^2}$ 1

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5(b)

$$\frac{s^2-6s-9}{(s^2+9)(s-3)} = \frac{As+B}{s^2+9} + \frac{C}{s-3}$$ 1

$$\Rightarrow s^2-6s-9 = (As+B)(s-3) + C(s^2+9) \quad --- \; *$$

$s=3$: $\Rightarrow -18 = C\cdot 18 \Rightarrow C = -1$ 1

$s=0$: $\Rightarrow -9 = -3B - 9 \Rightarrow B = 0$ 1