

(10)

$$7(a) \quad f(x) = \begin{cases} x, & 0 < x \leq \pi \\ 2\pi - x, & \pi < x \leq 2\pi \end{cases} \quad [0, 2\pi]$$

$$b_n = \frac{2}{2\pi} \int_0^{2\pi} f(x) \sin\left(\frac{n\pi x}{2\pi}\right) dx$$

$$= \frac{1}{\pi} \left\{ \int_0^{\pi} x \sin\left(\frac{nx}{2}\right) dx + 2\pi \int_{\pi}^{2\pi} \sin\left(\frac{nx}{2}\right) dx - \int_{\pi}^{2\pi} x \sin\left(\frac{nx}{2}\right) dx \right\} \quad 1$$

$$= \frac{1}{\pi} \left\{ \left[ \frac{4}{n^2} \sin\left(\frac{nx}{2}\right) - x \cdot \frac{2}{n} \cos\left(\frac{nx}{2}\right) \right]_0^{\pi} + 2\pi \left[ -\frac{2}{n} \cos\left(\frac{nx}{2}\right) \right]_{\pi}^{2\pi} - \left[ \frac{4}{n^2} \sin\left(\frac{nx}{2}\right) - x \cdot \frac{2}{n} \cos\left(\frac{nx}{2}\right) \right]_{\pi}^{2\pi} \right\} \quad 4$$

$$= \frac{1}{\pi} \left\{ \frac{4}{n^2} \sin\left(\frac{n\pi}{2}\right) - \frac{2\pi}{n} \cos\left(\frac{n\pi}{2}\right) - \frac{4\pi}{n} \cos(n\pi) + \frac{4\pi}{n} \cos\left(\frac{n\pi}{2}\right) - \frac{4}{n^2} \sin(n\pi) + \frac{4\pi}{n} \cos(n\pi) + \frac{4}{n^2} \sin\left(\frac{n\pi}{2}\right) - \frac{2\pi}{n} \cos\left(\frac{n\pi}{2}\right) \right\}$$

$$= \frac{1}{\pi} \left\{ \frac{8}{n^2} \sin\left(\frac{n\pi}{2}\right) - \frac{4}{n^2} \sin(n\pi) \right\} = \frac{8}{\pi n^2} \sin\left(\frac{n\pi}{2}\right) \quad 5$$

$$= \frac{8}{\pi n^2} (-1)^m \text{ if } n = 2m+1, \quad m = 1, 2, 3, \dots$$

$$= 0 \quad \text{otherwise}$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n^2} \sin\left(\frac{n\pi x}{2}\right)$$