

(19)

$$10(a) \quad f = u + iv; \quad u = x \sin x \cosh y - y \cos x \sinh y$$

$$\text{and } v = x \cos x \sinh y + y \sin x \cosh y.$$

$$\text{C.R. eqns} \quad u_x = v_y \quad \text{and} \quad u_y = -v_x$$

$$\text{1st eqn: } u_x \equiv \cosh y \{ \sin x + x \cos x \} + y \sinh y \sin x \quad 1$$

$$v_y = x \cos x \cosh y + \sin x \{ \cosh y + y \sinh y \} \quad 1$$

$$u_x - v_y \equiv \cosh y \{ \cancel{\sin x} + \cancel{x \cos x} - \cancel{x \cos x} - \cancel{\sin x} \} \\ + \sinh y \{ \cancel{y \sin x} - \cancel{y \sin x} \} \equiv 0 \quad 2$$

So 1st C-R eqn identically satisfied.

$$u_y = x \sin x \sinh y - \cos x \{ \sinh y + y \cosh y \} \quad 1$$

$$v_x = \sinh y \{ \cos x - x \sin x \} + y \cosh y \cos x \quad 1$$

$$\text{2nd eqn: } u_y + v_x \equiv \sinh y \{ \cancel{x \sin x} - \cancel{\cos x} + \cancel{\cos x} - \cancel{x \sin x} \} \\ + \cosh y \{ \cancel{y \cos x} - \cancel{y \cos x} \} \equiv 0 \quad 2$$

Hence C-R eqns are identically satisfied

and u, v , and all derivatives are continuous 1

everywhere. Thus f' exists everywhere 1

and therefore analytic everywhere.