$(1+x^2)y'' - 3xy' + 3y = 0$, $y(0)=2$, $y'(0) = -2$

$y = \sum_0 a_n x^n$, $y' = \sum_0 n a_n x^{n-1}$, $y'' = \sum_0 n(n-1)a_n x^{n-2}$

d.e. $\Rightarrow$ $\sum_0 n(n-1)a_n x^{n-2} + \sum_0 n(n-1)a_n x^n$

$-3\sum_0 n a_n x^n + 3\sum_0 a_n x^n = 0$ 1

shifts:

$\sum_0^\infty n(n-1)a_n x^{n-2} = \sum_0 (n+2)(n+1)a_{n+2}x^n$ 2

$\therefore$ d.e. becomes

$\sum_0 [(n+2)(n+1)a_{n+2} + n(n-1)a_n - 3na_n + 3a_n]x^n = 0$

$\Rightarrow$ $(n+2)(n+1)a_{n+2} + a_n[n^2 - n - 3n + 3] = 0$, $n = 0,1,2,\ldots$ 1

i.e. $\boxed{a_{n+2} = \frac{-a_n[n^2-4n+3]}{(n+2)(n+1)} = -a_n\frac{(n-3)(n-1)}{(n+2)(n+1)}}$ 3

$a_2 = -\frac{a_0(3)}{2} = -\frac{3}{2}a_0$

$a_3 = -\frac{a_1 \cdot 0}{3\cdot 2} = 0$; $a_4 = \frac{-a_2(-1)}{12} = \frac{1}{12}\cdot\frac{-3}{2}a_0$

$= -\frac{1}{8}a_0$

$a_5 = -\frac{a_3(0)}{20} = 0$ 3

All odd numbers are zero (except a_1)

$a_3 = a_5 = a_7 = \ldots = 0$ 1