7. (a). Determine the Fourier sine series of the function $f(x) = \begin{cases} x/\pi, & 0 < x \le \pi \\ 2 - x/\pi, & \pi < x \le 2\pi \end{cases}$ on the interval $[0, 2\pi]$.

(b). Apply the method of separation of variables to solve the boundary-value problem

$$\begin{aligned} 4\frac{\partial^2 u}{\partial x^2} &= \frac{\partial^2 u}{\partial t^2};\ 0 < x < 2\pi,\ t > 0. \\ u(0,t) &= u(2\pi, t) = 0,\ t > 0. \\ \frac{\partial u}{\partial t}(x,0) &= 0,\ 0 < x < 2\pi \\ u(x,0) &= \begin{cases} x/\pi, & 0 < x \le \pi \\ 2 - x/\pi, & \pi < x \le 2\pi \end{cases} \end{aligned}$$

[14+16=30 Marks]

8. (a). Determine all possible values of $(-3i)^{2i}$.

(b). Find all z such that $\sin(z) = 3i$

[7+7=14 Marks]

9. (a). Given that $f(z) = x\sin(x)\cosh(y) - y\cos(x)\sinh(y) + i[x\cos(x)\sinh(y) + y\sin(x)\cosh(y)]$

(i). Where does $f'(z)$ exist? (ii). Where is $f(z)$ analytic?

Justify your answers.

(b). Given that $u = x^3 + 2xy - 3y^2x - 1$, determine all conjugate harmonic functions v such that $g(z) = u + iv$ is analytic everywhere.

[10+7=17 Marks]