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**Attempt as many questions as possible.**
**Marks are awarded for working, as well as for answers.**
**A formula sheet is attached.**

1. (a) For the function $y = f(x) = |2x - 9|$ :
   - (i) write the domain and range of the function and sketch it;
   - (ii) show, using algebra, that $f(x)$ is not one-to-one;
   - (iii) find a restriction of the domain such that the function is one-to-one.

   (b) Given $y = f(x) = \sqrt{5x-6}$ for $x \geq \frac{6}{5}$:
   - (i) find $f^{-1}(x)$ ;
   - (ii) sketch $f(x)$ and $f^{-1}(x)$ on the same set of axes and label all intersections of the two functions.

   (c) Evaluate the following limits
   - (i) $\lim_{x \to \infty} \frac{4x^3 - 3x^2 + 2}{5 - x^3}$
   - (ii) $\lim_{x \to 3} \frac{x^2 + x - 12}{x^2 - 4x + 3}$
   - (iii) $\lim_{x \to 1} \frac{3x^4 + 2x^2 - 6x + 1}{x^5 - 1}$
   - (iv) $\lim_{x \to 0} \frac{4\cos x - e^{5x} - 3}{x - 2\sin x}$.

[8+7+12 = 27 Marks]

2. (a) Find $\frac{dy}{dx}$ in the following cases:
   - (i) $y = \frac{7x-1}{5x+3}$
   - (ii) $y = (x\cos x - \sin x)^3$
   - (iii) $y = (x^2 - x + 5)e^{2x}$
   - (iv) $3x^4 - 5xy - 2y^3 = 10$.

   (b) Use logarithmic differentiation to find $\frac{dy}{dx}$ if $y = \frac{(4x^2-3)^{5/8}}{(2x^3 - 6x + 3)^{1/6}}$

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2. (c) Find $\dfrac{dy}{dx}$ in the following cases:

(i) $y = \tan^{-1}(2\sqrt{x})$ (ii) $y = (4x^3 + 2)\cosh x$

(iii) $y = \displaystyle\int_4^x \frac{\ln t - 5\sin t}{e^{4t} + 1}\, dt.$

[13+5+7=25 Marks]

3. (a) (i) State the definitions of $\cosh x$ and $\sinh x$.

(ii) Using the definitions in (i), prove that $\ln(\cosh x - \sinh x) = -x$.

(b) Find the absolute maximum and absolute minimum of $y = f(x) = 2x^3 + 3x^2 - 12x + 5$ for $-3 \le x \le 3$.

(c) Sketch $y = f(x) = \dfrac{8(x^2 - 1)}{(x-2)^2}$ after examining:

(i) domain; (ii) vertical asymptotes;
(iii) symmetry; (iv) intercepts;
(v) behaviour as $x \to \pm\infty$; (vi) sign of $y$.

[6+5+11=22 Marks]

4. (a) Find

(i) $I = \displaystyle\int \frac{x^2}{(2x^3 + 5)^2}\, dx$

(ii) $I = \displaystyle\int (6x + 7)e^{6x}\, dx$

(iii) $I = \displaystyle\int \frac{\sin x}{\sqrt{5 - 2\cos x}}\, dx.$

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4. (b) Use a standard integral to evaluate $I = \int_0^8 \frac{1}{\sqrt{x^2 + 36}}\,dx$, and express the answer in terms of the natural logarithm.

(c) Use partial fractions to find $I = \int \frac{3x - 5}{x^2 + 2x - 3}\,dx$.

[12+4+5 = 21 marks]

5. (a) (i) Convert $I = \int_{\sqrt{2}}^{2} \frac{2x}{\sqrt{16 - x^4}}dx$ to a standard integral by making the substitution $u = x^2$.

(ii) Use the standard integral obtained in (i) to evaluate *I*, and express the answer in terms of $\pi$.

(b) Sketch, and find the finite area bounded by the curve $y = x^2 - 2x - 3$ , and the line $y = -(1 + x)$.

(c) Find the volume formed when the area below the curve $y = (4 - x^2)\sqrt{x}$, and above the $x$ axis, is rotated about the $x$ axis.

[6+8+6=20 Marks]

6. (a) Solve

(i) $\frac{dy}{dx} = 16xy^{3/4}; \quad y(2) = 1$

(ii) $\frac{dy}{dx} + \frac{y}{x} = 4x^2 - 9x + 4; \quad y(1) = 2$.

(b) (i) Find the first four terms of the sequence $\{a_n\}$ with $n^{th}$ term

$$a_n = \frac{6n + (-1)^n}{n}.$$

(ii) Find $\lim_{n \to \infty} a_n$ (if it exists).

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6. (c) Find the open interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{(x-6)^n}{(n-1)2^n}.$$

[13+5+5 = 23 Marks]

7. (a) (i) Derive the first <u>three</u> terms of the MacLaurin series for

$f(x) = (1+x)^{-1/3}$.

(ii) Hence approximate $(1.06)^{-1/3}$ to 4 decimal places.

(b) Given that $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots$ for $-\infty < x < \infty$:

(i) write down the first three non-zero terms of the power series for $\frac{x - \sin x}{x^3}$;

(ii) use the first three non-zero terms of the series for $\frac{x - \sin x}{x^3}$ to approximate

$$I = \int_0^1 \frac{x - \sin x}{x^3}\, dx.$$

(c) Given $\underset{\sim}{A} = 4\underset{\sim}{i} - 8\underset{\sim}{j} + \underset{\sim}{k}$, and $\underset{\sim}{B} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, find:

(i) $|\underset{\sim}{A}|$ and $|\underset{\sim}{B}|$;

(ii) $\underset{\sim}{A} \cdot \underset{\sim}{B}$;

(iii) the cosine of the angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$;

(iv) $\underset{\sim}{A} \times \underset{\sim}{B}$.

[6+7+8=21 Marks]

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8. (a) (i) Find a vector perpendicular to the plane containing the points $P(0, -3, 4)$, $Q(2, -1, 7)$, and $R(0, -5, 7)$.

(ii) Hence find the area of the triangle PQR.

(iii) Find the equation of the plane containing the points P, Q, and R.

(b) Find parametric equations of the line through the points $P(5, -1, 3)$ and $Q(9, 4, -3)$.

(c) Find the point at which the line

$$x = 4 - 3t,\ y = -3 + 7t,\ z = 1 + 4t$$

intersects the plane $5x + 2y - z = 23$.

(d) Find the volume of the box whose edges are determined by the vectors $\underset{\sim}{A} = 3\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{B} = 4\underset{\sim}{i} - 2\underset{\sim}{j} + 7\underset{\sim}{k}$, and $\underset{\sim}{C} = 3\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}$.

[8+4+5+4=21 Marks]

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# List of Standard Integrals and Trigonometric Formulae

**Standard Integrals (+C omitted)**

| Function | Integral |
|---|---|
| 1 $1/(a^2 - x^2)$ | $\frac{1}{a}\tanh^{-1}(x/a)$ or $\frac{1}{2a}\ln\frac{a+x}{a-x}$ if $\lvert x\rvert < a$ |
| 2 $1/(x^2 - a^2)$ | $-\frac{1}{a}\coth^{-1}(x/a)$ or $\frac{1}{2a}\ln\frac{x-a}{x+a}$ if $\lvert x\rvert > a$ |
| 3 $1/(x^2 + a^2)$ | $\frac{1}{a}\tan^{-1}(x/a)$ |
| 4 $1/\sqrt{a^2 - x^2}$ | $\sin^{-1}(x/a)$ |
| 5 $1/\sqrt{x^2 - a^2}$ | $\cosh^{-1}(x/a)$ or $\ln\{x + \sqrt{x^2 - a^2}\}$ if $x > a$ |
| | $-\cosh^{-1}(-x/a)$ or $\ln\{-x + \sqrt{x^2 - a^2}\}$ if $x < -a$ |
| 6 $1/\sqrt{x^2 + a^2}$ | $\sinh^{-1}(x/a)$ or $\ln\{x + \sqrt{x^2 + a^2}\}$ |
| 7 $\sqrt{a^2 - x^2}$ | $\frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\sin^{-1}(x/a)$ |
| 8 $\sqrt{x^2 - a^2}$ | $\frac{1}{2}x\sqrt{x^2 - a^2} - \frac{1}{2}a^2\cosh^{-1}(x/a)$ if $x \ge a$ |
| | $\frac{1}{2}x\sqrt{x^2 - a^2} + \frac{1}{2}a^2\cosh^{-1}(-x/a)$ if $x \le -a$ |
| 9 $\sqrt{x^2 + a^2}$ | $\frac{1}{2}x\sqrt{x^2 + a^2} + \frac{1}{2}a^2\sinh^{-1}(x/a)$ |
| 10 $e^{ax}\sin bx$ | $\frac{e^{ax}(a\sin bx - b\cos bx)}{a^2 + b^2}$ |
| 11 $e^{ax}\cos bx$ | $\frac{e^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2}$ |

**Reduction Formulae**

12
$$\int \sin^m x\cos^n x\,dx = \frac{\sin^{m+1}x\cos^{n-1}x}{m+n} + \frac{n-1}{m+n}\int \sin^m x\cos^{n-2}x\,dx$$
$$\text{or } -\frac{\sin^{m-1}x\cos^{n+1}x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2}x\cos^n x\,dx$$

13
$$\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$

14
$$\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$$

**Trigonometric Formulae**

$$\sin(x+y) = \sin x\,\cos y + \cos x\,\sin y \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$
$$\cos(x+y) = \cos x\,\cos y - \sin x\,\sin y \qquad \sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x\,\tan y} \qquad \sin x\cos x = \tfrac{1}{2}\sin 2x$$